Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 298 (2006) 298–306
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Adaptive sliding mode control for synchronization of chaotic gyros with fully unknown parameters

Jun-Juh Yan[a,*], Meei-Ling Hung[b,c], Teh-Lu Liao[b]

[a]*Department of Computer and Communication, Shu-Te University, Kaohsiung 824, Taiwan, ROC*
[b]*Department of Engineering Science, National Cheng Kung University, Tainan 701, Taiwan, ROC*
[c]*Department of Electrical Engineering, Far-East College, Tainan 744, Taiwan, ROC*

Received 12 February 2006; received in revised form 6 May 2006; accepted 20 May 2006
Available online 18 July 2006

Abstract

This study addresses the synchronization of chaotic gyros with unknown parameters and external disturbance via adaptive sliding mode control. To achieve synchronization, a switching surface is adopted such that it becomes easy to ensure the stability of the error dynamics in the sliding mode. Then an adaptive sliding mode controller (ASMC) is derived to guarantee the occurrence of the sliding motion even when the parameters of the drive and response gyros are fully unknown. Numerical simulations are presented to verify that the synchronization can be achieved by using this ASMC.

1. Introduction

In the last decade, control and synchronization of chaotic systems have become an important topic since the pioneering work of Pecora and Carroll [1]. Chaos synchronization can be applied in the vast areas of physics and engineering systems such as in chemical reactions, power converters, biological systems, information processing, especially in secure communication [2–6]. Many different chaos synchronization strategies have been developed, such as impulsive control [7,8], adaptive control [9,10], variable structure control [11–13], optimal control [14], digital redesign control [15], backstepping control [16,17], and so on.

On the other hand, gyro dynamics is considered to be one of the most important problems and has been studied by many authors. The gyro has attributes of great utility to navigational, aeronautical and space engineering [18]. In the past years, the gyros have been found with rich phenomena and give benefit for understanding of gyro systems. Different types of gyros with linear/nonlinear damping are investigated for predicting the dynamic responses such as periodic and non-periodic (chaotic) motions [19–22]. Recently, based on active control technique, Lei et al. [21] used two control inputs to achieve synchronization for chaotic gyros. The controller was synthesized based on fully known parameters. In real-life applications, however, the gyro's parameters are inevitably perturbed by external inartificial factors and cannot be exactly known in

*Corresponding author. Tel.: +886 7 6158000x4806; fax: +886 7 6158000x4899.
E-mail address: jjyan@mail.stu.edu.tw (J.-J. Yan).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.05.017

advance. Therefore, it is highly desirable to propose a new synchronization controller for chaotic gyros to release the limitation of knowing the system parameters in advance.

The purpose of this paper lies in the development of an adaptive sliding mode control (ASMC) for synchronizing the state trajectories of two chaotic gyros with unknown parameters and external disturbance. A switching surface, which makes it easy to guarantee the stability of the error dynamics in the sliding mode, is first proposed. And then, based on this switching surface, an ASMC is derived to guarantee the occurrence of the sliding motion. In particular, the limitations of knowing system parameters and external disturbance in priori are also released due to the adaptive scheme. Furthermore, compared with the work of Lei et al. [21], this approach needs only a single controller to realize synchronization, which has important significance in reducing the cost and complexity for controller implementation. Finally, we present the numerical simulation results to illustrate the effectiveness of the proposed ASMC scheme.

This paper is organized as follows: Section 2 describes the dynamics of a nonlinear gyro. In Section 3, the synchronization problem for chaotic gyros is formulated and the stable switching surface is first derived. Then a novel ASMC is designed to guarantee the occurrence of the sliding mode. Numerical simulations that confirm the validity and feasibility of the proposed method are shown in Section 4. Finally, conclusions are presented in Section 5.

Throughout this paper, it is noted that, $|w|$ represents the absolute value of w and $\|w\|$ represents the Euclidean norm when w is a vector. sign(s) is the sign function of s, if $s>0$, sign(s) $=1$; if $s=0$, sign(s) $=0$; if $s<0$, sign(s) $=-1$.

2. Dynamics of a symmetric gyro with nonlinear damping

In this paper, a symmetric gyro with linear-plus-cubic damping, see Fig. 1 in Ref. [18], is considered. The equation governing the motion of this symmetric gyro in terms of the angle θ is given by [18]

$$\ddot{\theta}+\alpha^2\frac{(1-\cos\,\theta)^2}{\sin^3\theta}+c_1\dot{\theta}+c_2\dot{\theta}^3-\beta\,\sin\,\theta=f\,\sin\,wt\,\sin\,\theta, \tag{1}$$

where $f\sin wt$ is a parametric excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are linear and nonlinear damping terms, respectively, and $\alpha^2[(1-\cos\,\theta)^2/\sin^3\theta]-\beta\,\sin\,\theta$ is a nonlinear resilience terms. The normalized equations in convenient

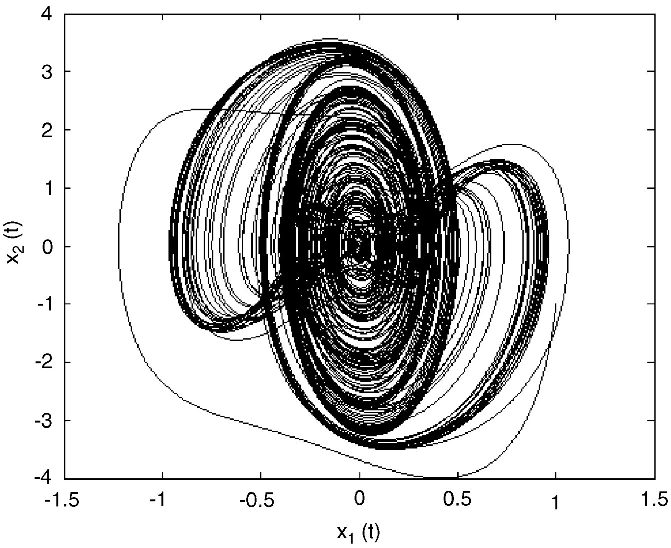

Fig. 1. The phase plane of x_1 versus x_2.

first-order form are

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x_2^3 + \beta \sin x_1 + f \sin wt \sin x_1, \tag{2}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$.

The dynamics of this system has been extensively studied in [18,19] for a space range of the amplitude of the term f. In particular, for the parameter value of $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $w = 2$ and $f = 35.5$, this symmetric gyro displays chaotic behavior [18]. Fig. 1 shows the chaotic attractor with initial condition of $[x_1(0), x_2(0)] = [-1, 2]$. In the following, we will consider the synchronization of two identical gyros and give an explicit and simple procedure to establish an ASMC to guarantee the synchronization even when the system's parameters are fully unknown.

3. Synchronization problem formulation and ASMC design

Let us consider the following two nonlinear gyros, where the drive system and response system are denoted with x and y, respectively. The systems are

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x_2^3 + \beta \sin x_1 + f \sin wt \sin x_1 \tag{3}$$

and

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = -\alpha^2 \frac{(1-\cos y_1)^2}{\sin^3 y_1} - c_1 y_2 - c_2 y_2^3 + \beta \sin y_1 + f \sin wt \sin y_1 + \rho(t) + u(t). \tag{4}$$

To generally describe the gyros in the true physical world, the response gyro (4) is assumed to be subject to external disturbance $\rho(t) \in R$. Without loss of generality, the external disturbance is bounded, i.e. $|\rho(t)| \leqslant \delta \in R^+$. We have introduced the single control input u into the second equation in the system (4). This single input is to be determined for the purpose of synchronizing the two nonlinear gyros with the same but unknown parameters α, β, c_1, c_2, w, f and the unknown disturbance bound δ. Let us define the synchronization errors between the response system (4) and the drive system (3) as follows:

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \tag{5}$$

then the dynamics of the error system is determined, directly subtracting (3) from (4), as follows:

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = -c_1 e_2 + \alpha^2 g(x_1, y_1) - c_2 y_2^3 + c_2 x_2^3 + (\beta + f \sin wt)(\sin y_1 - \sin x_1) + \rho(t) + u, \tag{6}$$

where

$$g(x_1, y_1) = \frac{(1-\cos x_1)^2}{\sin^3 x_1} - \frac{(1-\cos y_1)^2}{\sin^3 y_1}.$$

It is clear that the synchronization problem is replaced by the equivalent of stabilizing the error dynamics (6) using a suitable choice of the control scheme u. Therefore, the considered goal of this paper is that for any given nonlinear gyros as (3) and (4), an ASMC is designed such that the asymptotical stability of the resulting error system (6) can be achieved in the sense that $\|e(t)\| \to 0$ as $t \to \infty$, where $e(t) = [e_1, e_2]$.

As a sequence, to achieve the synchronization via ADMC, two basic steps are involved: (1) selecting an appropriate switching surface such that the sliding motion on the sliding mode is stable and ensures $\lim_{t\to\infty} \|e(t)\| = 0$; and (2) establishing an ASMC law which guarantees the existence of the sliding mode $s(t) = 0$ even with fully unknown system's parameters.

To ensure the asymptotical stability of the sliding mode, a switching surface $s(t)$ in the error space is defined as follows:

$$s(t) = e_2(t) + \lambda e_1(t), \tag{7}$$

where $s(t) \in R$ and λ is design parameter which can be easily determined later. As well known, when the system operates in the sliding mode, it satisfies the following equations [23,24]

$$s(t) = e_2(t) + ke_1(t) = 0. \tag{8}$$

Therefore, the following sliding mode dynamics can be obtained as

$$\dot{e}_1(t) = e_2(t) = -\lambda e_1(t), \tag{9}$$

$$\dot{e}_2 = -c_1 e_2 + \alpha^2 g(x_1, y_1) - c_2 y_2^3 + c_2 x_2^3 + (\beta + f \sin wt)(\sin y_1 - \sin x_1) + \rho(t) + u. \tag{10}$$

Obviously, if the design parameter $\lambda > 0$ is specified, the stability of (9) is surely guaranteed, that is $\lim_{t\to\infty} e_1(t) = 0$. Furthermore, by Eq. (8), $e_2(t)$is also stable, that is $\lim_{t\to\infty} e_2(t) = 0$. Meanwhile, it is worthy of note that the value λ is also relative to the speed of error system response in the sliding mode.

Having established an appropriate switching surface, the next step is to design an ASMC scheme to drive the error system trajectories onto the switching surface $s(t) = 0$. To ensure the occurrence of the sliding mode, an ASMC scheme is proposed as

$$u(t) = -\lambda e_2 - \gamma \xi \, \mathrm{sign}(s), \tag{11}$$

where $\gamma > 1$, $\xi = \hat{c}_1 |e_2| + \hat{\alpha} |g(x_1, y_1)| + \hat{c}_2 |y_2^3 - x_2^3| + \hat{\eta} |\sin y_1 - \sin x_1| + \hat{\delta}$. The adaptive laws are

$$\begin{aligned}
\dot{\hat{c}}_1 &= |e_2||s|, \quad \hat{c}_1(0) = \hat{c}_{10}, \\
\dot{\hat{c}}_2 &= |y_2^3 - x_2^3||s|, \quad \hat{c}_2(0) = \hat{c}_{20}, \\
\dot{\hat{\alpha}} &= |g(x_1, y_1)||s|; \ \hat{\alpha}(0) = \hat{\alpha}_0, \\
\dot{\hat{\eta}} &= |\sin y_1 - \sin x_1||s|, \quad \hat{\eta}(0) = \hat{\eta}_0, \\
\dot{\hat{\delta}} &= |s|, \quad \hat{\delta}(0) = \hat{\delta}_0,
\end{aligned} \tag{12}$$

where $\hat{c}_{10}$, $\hat{c}_{20}$, $\hat{\alpha}_0$, $\hat{\eta}_0$ and $\hat{\delta}_0$ are the positive and bounded initial values of $\hat{c}_1$, $\hat{c}_2$, $\hat{\alpha}$, $\hat{\eta}$ and $\hat{\delta}$, respectively.

The proposed adaptive control scheme above will guarantee the globally asymptotical stability for the error system (6), and is proven in the following theorem.

Theorem 1. *Consider the error dynamics* (6)*, if this system is controlled by* $u(t)$ *in* (11) *with adaptation law* (12)*. Then the system trajectory converges to the sliding surface* $s(t) = 0$.

Before proving Theorem 1, the Barbalat's lemma is given below.

Lemma 1. (Barbalat's lemma [25]). *If* $w : R \to R$ *is a uniformly continuous function for* $t \geqslant 0$ *and if* $\lim_{t\to\infty} \int_0^t |w(\lambda)| \, \mathrm{d}\lambda$ *exists and is finite, then* $\lim_{t\to\infty} w(t) = 0$.

After introducing Lemma 1, we are ready to prove Theorem 1.

Proof of Theorem 1. Let

$$\theta_1 = \hat{c}_1 - |c_1|, \ \theta_2 = \hat{c}_2 - |c_2|, \ \theta_3 = \hat{\alpha} - \alpha^2, \ \theta_4 = \hat{\eta} - \eta \quad \text{and} \quad \theta_5 = \hat{\delta} - \delta, \tag{13}$$

where $\eta = |\beta| + |f|$. It is assumed that $|c_1|$, $|c_2|$, α^2, η and δ are unknown constants. Thus the following expression holds.

$$\dot{\theta}_1 = \dot{\hat{c}}_1, \ \dot{\theta}_2 = \dot{\hat{c}}_2, \ \dot{\theta}_3 = \dot{\hat{\alpha}}, \ \dot{\theta}_4 = \dot{\hat{\eta}} \quad \text{and} \quad \dot{\theta}_5 = \dot{\hat{\delta}}. \tag{14}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\left(s^2 + \sum_{i=1}^{5} \theta_i^2\right). \tag{15}$$

It is clear that V is a positive definite function, then taking the derivative of $V(t)$ with respect to time t, one has

$$\dot{V}(t) = s\dot{s} + \sum_{i=1}^{5} \theta_i \dot{\theta}_i. \tag{16}$$

Introducing (6), (7), (11) and (12) into (16) yields

$$\begin{aligned}
\dot{V}(t) &= s[\dot{e}_2 + \lambda \dot{e}_1] + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \\
&= s(\dot{e}_2 + \lambda e_2) + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \\
&= s(-c_1 e_2 + \alpha^2 g(x_1, y_1) - c_2(y_2^3 - x_2^3) + (\beta + f \sin wt)(\sin y_1 - \sin x_1) \\
&\quad + \rho(t)) - \gamma \xi s \,\mathrm{sign}(s) + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \\
&\leqslant |c_1||e_2||s| + \alpha^2 |g(x_1, y_1)||s| + |c_2||y_2^3 - x_2^3||s| + \overbrace{(|\beta| + |f|)}^{\eta} |\sin y_1 \\
&\quad - \sin x_1||s| + \delta|s| - \gamma \xi |s| + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \\
&= \underbrace{(|c_1| - \hat{c}_1)}_{-\theta_1} |e_2||s| + \underbrace{(\alpha^2 - \hat{\alpha})}_{-\theta_2} |g(x_1, y_1)||s| + \underbrace{(|c_2| - \hat{c}_2)}_{-\theta_3} |y_2^3 - x_2^3||s| \\
&\quad + \underbrace{(\eta - \hat{\eta})}_{-\theta_4} |\sin y_1 - \sin x_1||s| + \underbrace{(\delta - \hat{\delta})}_{-\theta_5} |s| + \xi|s| - \gamma \xi |s| + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \\
&= (1 - \gamma)\xi|s|
\end{aligned} \tag{17}$$

Since $\gamma > 1$ has been specified in (11), we obtain the following inequality

$$\dot{V}(t) \leqslant -(\gamma - 1)\xi|s|. \tag{18}$$

Now if we define $|w(t)| = (\gamma - 1)\xi|s| > 0$, and integrating the above equation from zero to t, it yields

$$V(t) \leqslant V(0) - \int_0^t |w(\tau)| \,\mathrm{d}\tau \Rightarrow V(0) \geqslant V(t) + \int_0^t |w(\tau)| \,\mathrm{d}\tau \geqslant \int_0^t |w(\tau)| \,\mathrm{d}\tau. \tag{19}$$

Taking the limit as $t \to \infty$ on both side of (19) gives

$$\infty > V(0) \geqslant \lim_{t \to \infty} \int_0^t |w(\tau)| \,\mathrm{d}\tau. \tag{20}$$

Thus according to Barbalat's lemma (see Lemma 1), we obtain

$$\lim_{t \to \infty} |w(t)| = \lim_{t \to \infty} (\gamma - 1)\xi|s| \to 0. \tag{21}$$

Since $(\gamma - 1) > 0$ and $\xi > 0$ for all $t > 0$, (21) implies $s(t) \to 0$ as $t \to \infty$. Hence the proof is achieved completely. □

The following theorem is introduced to guarantee the asymptotical stability of the closed-loop error system (6).

Theorem 2. *The closed-loop error system* (6) *driven by the controller* $u(t)$ (11) *with adaptation law* (12) *is asymptotically stable in the large.*

Proof. When the error system (6) is driven by the control input $u(t)$ given in (11) with adaptation law (12), the trajectory of the error dynamics system converges to the sliding mode $s = 0$, as previously discussed in

Theorem 1. Thus the equivalent error dynamics system in the sliding mode is obtained as shown in (9). As discussed previously, in (7), the values of $\lambda > 0$ is specified to guarantee the asymptotical stability of the error dynamic system. Consequently, the asymptotical stability of the closed-loop error system is also ensured. The theorem is therefore proved. □

Remark 1. The controller in (11) demonstrates a discontinuous control law and the phenomenon of chattering will appear. In order to eliminate the chattering, the controller (11) can be modified as

$$u(t) = -\lambda e_2 - \gamma \xi \frac{s}{|s| + \varepsilon}, \tag{22}$$

where ε is a sufficiently small positive constant. From the works [26,27], the solution of system (6) with (11) can be made arbitrarily close to the solution (6) with (22), if one chooses ε sufficiently small.

4. Numerical example

In this section, simulation results are presented to demonstrate the effectiveness of the proposed adaptive synchronization algorithm. All the simulation procedures are coded and executed using the software of MATLAB. The system parameters are chosen as follows: $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $w = 2, f = 35.5$ and the external disturbance in response system (4) is defined as $\rho(t) = 0.2 \cos 2t$. Thus $|\rho(t)| \leqslant 0.2 = \delta$ can be obtained. The initial states of the drive system (3) are $x_1(0) = 0.5$, $x_2(0) = 1$ and initial states of the response system (4) are $y_1(0) = 1$, $y_2(0) = 2$.

As mentioned in Section 3, the proposed design procedure may be obtained as follows:

Step 1: According to (7), we select $\lambda = 1 > 0$ to result in a stable sliding mode. Therefore the switching surface equation is

$$s(t) = e_2 + e_1. \tag{23}$$

Step 2: From (22), the continuous control input is determined as

$$u(t) = -e_2 - \gamma\left(\hat{c}_1|e_2| + \hat{\alpha}|g(x_1, y_1)| + \hat{c}_2|y_2^3 - x_2^3| + \hat{\eta}|\sin y_1 - \sin x_1| + \hat{\delta}\right)\frac{s}{|s| + 0.01} \tag{24}$$

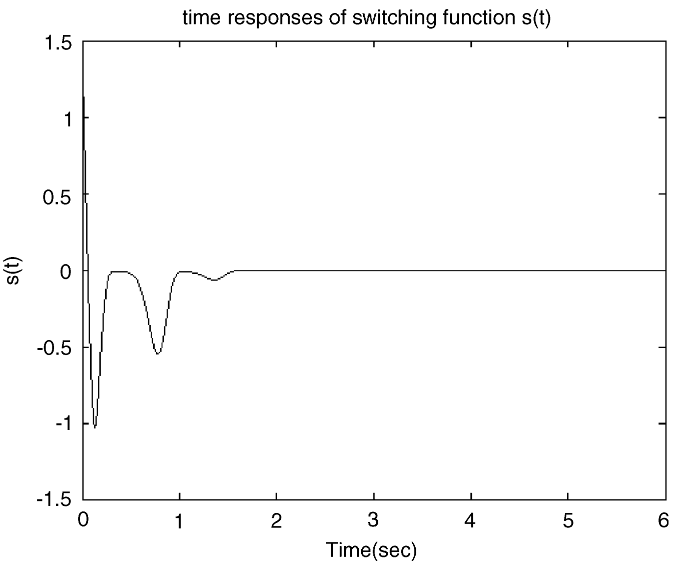

Fig. 2. Time response of $s(t)$.

with $\gamma = 1.1 > 1$ and the adaptive laws are

$$\begin{aligned} \dot{\hat{c}}_1 &= |e_2||s|, \quad \hat{c}_1(0) = 0.9, \\ \dot{\hat{c}}_2 &= |y_2^3 - x_2^3||s|, \quad \hat{c}_2(0) = 0.7, \\ \dot{\hat{\alpha}} &= |g(x_1, y_1)||s|, \quad \hat{\alpha}(0) = 0.5, \\ \dot{\hat{\eta}} &= |\sin y_1 - \sin x_1||s|, \quad \hat{\eta}(0) = 0.3, \\ \dot{\hat{\delta}} &= |s|, \quad \hat{\delta}(0) = 0.1. \end{aligned} \tag{25}$$

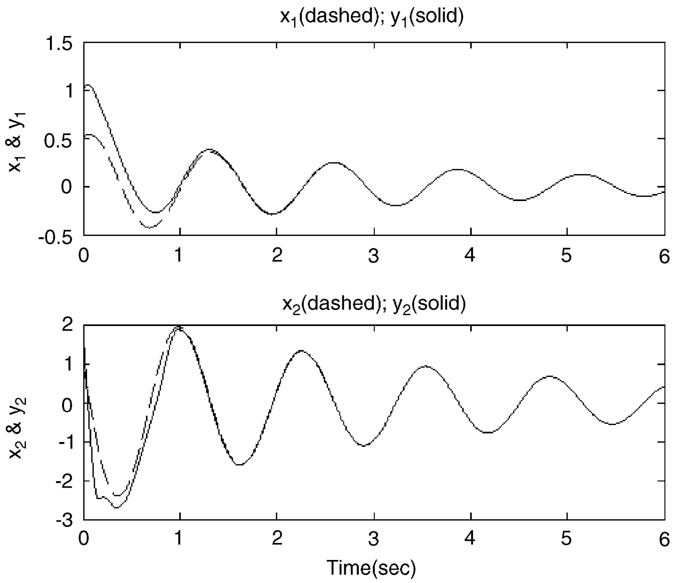

Fig. 3. State responses for the controlled drive-response gyros.

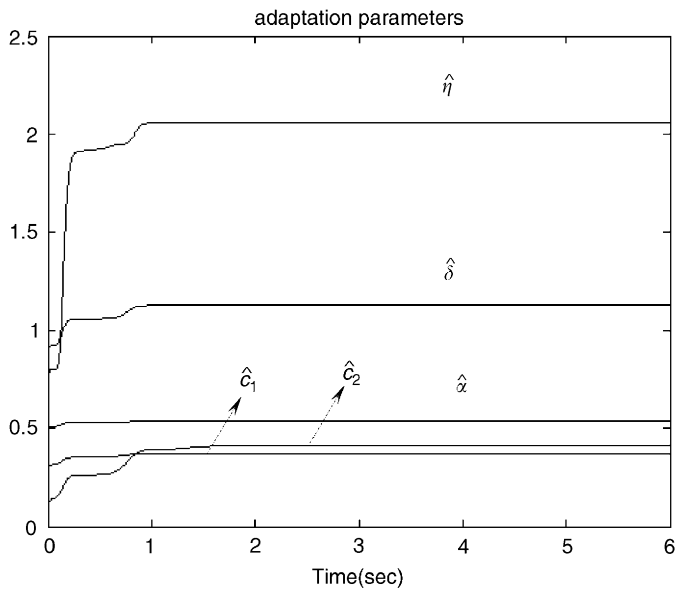

Fig. 4. Time responses of adaptation parameters.

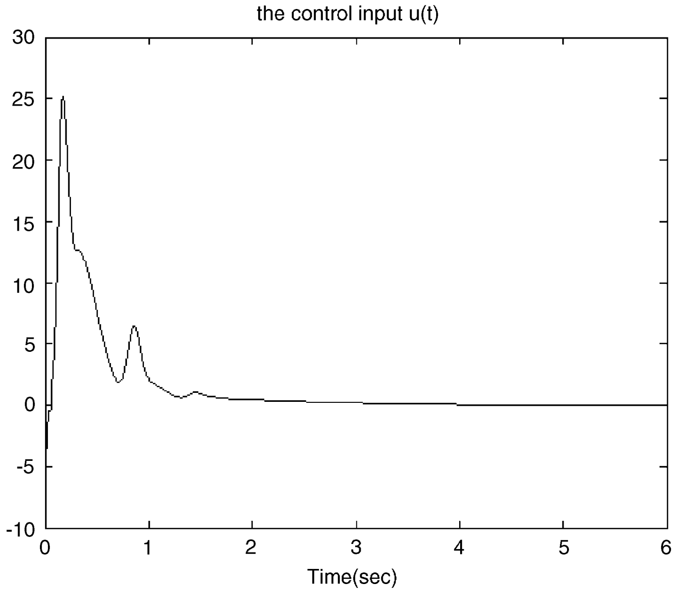

Fig. 5. Time responses of control input.

The simulation results are shown in Figs. 2–5 under the proposed ASMC (24) with the adaptation algorithm (25). Figs. 2 and 3 show, respectively, the corresponding $s(t)$ and state responses for the controlled drive-response gyros. The time responses of adaptation parameters and control input are shown in Figs. 4 and 5, respectively. From the simulation results, it is shown that the trajectory of error dynamics converges to $s = 0$ and the synchronization error also converges to zero. Thus the proposed ASMC works well and two chaotic nonlinear gyros from different initial values are indeed achieving chaos synchronization even when the system's parameters and external disturbance are fully unknown. Also the chattering does not appear due to the continuous control.

5. Conclusions

In this paper, adaptive synchronization control for chaotic symmetric gyros with linear-plus-cubic damping is demonstrated. A newly developed adaptive sliding mode controller has been proposed to cope with the fully unknown system parameters and external disturbance. Numerical simulations have verified the effectiveness of the proposed method.

References

[1] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, *Physical Review Letters* 64 (1990) 821–824.
[2] T. Yang, L.O. Chua, Secure communication via chaotic parameter modulation, *IEEE Transactions on Circuits and Systems I.* 43 (1996) 817–819.
[3] T.L. Liao, S.H. Tsai, Adaptive synchronization of chaotic systems and its application to secure communications, *Chaos Solitons and Fractals* 11 (2000) 1387–1396.
[4] M. Feki, An adaptive chaos synchronization scheme applied to secure communication, *Chaos Solitons and Fractals* 18 (2003) 141–148.
[5] M. Chen, D. Zhou, Y. Shang, A new observer-based synchronization scheme for private communication, *Chaos Solitons and Fractals* 24 (2005) 1025–1030.
[6] S.C. Lo, H.J. Cho, Chaos and control of discrete dynamic traffic model, *Journal of the Franklin Institute* 342 (2005) 839–851.
[7] J.T. Sun, Y.P. Zhang, Q.D. Wu, Impulsive control for the stabilization and synchronization of Lorenz systems, *Physics Letters A* 298 (2002) 153–160.
[8] S.H. Chen, Q. Yang, C.P. wang, Impulsive control and synchronization of unified chaotic system, *Chaos Solitons and Fractals* 20 (2004) 751–758.
[9] R. Schmitz, Use of chaotic dynamical systems in cryptography, *Journal of the Franklin Institute* 338 (2001) 429–441.

[10] C. Wang, S.S. Ge, Synchronization of two uncertain chaotic systems via adaptive backstepping, *International Journal of Bifurcation and Chaos* 11 (2001) 1743–1751.

[11] X. Yu, Y. Song, Chaos synchronization via controlling partial state of chaotic systems, *International Journal of Bifurcation and Chaos* 11 (2001) 1737–1741.

[12] M.J. Jang, C.C. Chen, C.O. Chen, Sliding mode control of chaos in the cubic Chua's circuit system, *International Journal of Bifurcation and Chaos* 12 (2002) 1437–1449.

[13] H.T. Yau, Design of adaptive sliding mode controller for chaos synchronization with uncertainties, *Chaos Solitons and Fractals* 22 (2004) 341–347.

[14] Y.P. Tian, X. Yu, Stabilization unstable periodic orbits of chaotic systems via an optimal principle, *Journal of the Franklin Institute—Engineering and Applied Mathematics* 337 (2000) 771–779.

[15] S.M. Guo, L.S. Shieh, G. Chen, C.F. Lin, Effective chaotic orbit tracker: a prediction-based digital redesign approach, *IEEE Transactions on Circuits and Systems I* 47 (2000) 1557–1560.

[16] T. Wu, M.S. Chen, Chaos control of the modified Chua's circuit system, *Physica D* 164 (2002) 53–58.

[17] J. Zhang, C. Li, H. Zhang, J. Yu, Chaos synchronization using single variable feedback based on backstepping method, *Chaos Solitons and Fractals* 21 (2004) 1183–1193.

[18] H.K. Chen, Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping, *Journal of Sound and Vibration* 255 (2002) 719–740.

[19] R.V. Dooren, Comments on "Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping", *Journal of Sound and Vibration* 268 (2003) 632–634.

[20] Z.M. Ge, H.K. Chen, Bifurcations and chaos in a rate gyro with harmonic excitation, *Journal of Sound and Vibration* 194 (1996) 107–117.

[21] Y. Lei, W. Xu, H. Zheng, Synchronization of two chaotic nonlinear gyros using active control, *Physics Letters A* 343 (2005) 153–158.

[22] X. Tong, N. Mrad, Chaotic motion of a symmetric gyro subjected to a harmonic base excitation, *Transactions of the American Society for Mechanical Engineering, Journal of Applied Mechanics* 68 (2001) 681–684.

[23] U. Itkis, *Control System of Variable Structure*, Wiley, New York, 1976.

[24] V.I. Utkin, *Sliding Mode and Their Applications in Variable Structure Systems*, Mir Editors, Moscow, 1978.

[25] V.M. Popov, *Hyperstability of Control System*, Springer, Berlin, 1973.

[26] C. Edwards, S.K. Spurgeon, R.J. Patton, Sliding mode observers for fault detection and isolation, *Automatica* 36 (2000) 541–548.

[27] J.E. Slotine, W. Li, *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, NJ, 1990.